CHAPTER 3

# Differential Equations

## Recap Notes

### DIFFERENTIAL EQUATION

An equation involving an independent variable, a dependent variable and the derivatives of the dependent variable is called differential equation.

- A differential equation involving derivatives of the dependent variable with respect to only one independent variable is called an ordinary differential equation.
- A differential equation involving derivatives with respect to more than one independent variables is called a partial differential equation.

### Order and Degree of a Differential Equation

- The order of highest derivative appearing in a differential equation is called order of the differential equation.
- The power of the highest order derivative appearing in a differential equation, after it is made free from radicals and fractions, is called degree of the differential equation.

**Note :** Order and degree (if defined) of a differential equation are always positive integers.

### HOMOGENEOUS DIFFERENTIAL EQUATIONS

- A differential equation of the form

$$\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$$

where, $f(x, y)$ and $g(x, y)$ are homogeneous functions of $x$ and $y$ of the same degree.

### LINEAR DIFFERENTIAL EQUATIONS

An equation of the form $\frac{dy}{dx}+Py=Q$ where $P$ and $Q$ are functions of $x$ only (or constants) is called a linear differential equation of the first order.

### SOLUTION OF A DIFFERENTIAL EQUATION

- Solution of a differential equation is a function of the form $y = f(x) + C$ which satisfies the given differential equation.
- **General Solution :** The solution of a differential equation which contains a number of arbitrary constants equal to the order of the differential equation.
- **Particular Solution :** A solution obtained by giving particular values to arbitrary constants in the general solution.

### METHODS OF SOLVING DIFFERENTIAL EQUATIONS

- **Equation in Variable Separable Form :** If the differential equation is of the form $f(x)\, dx = g(y)\, dy$, then the variables are separable and such equations can be solved by integrating on both sides. The solution is given by
  $\int f(x)\, dx=\int g(y)\, dy+C$, where $C$ is an arbitrary constant.
- **Equation Reducible to Homogeneous Form :** If the equation is of the form $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$, where $f(x, y)$ and $g(x, y)$ are homogeneous functions of the same degree in $x$ and $y$, then put $y = vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$ so that the dependent variable $y$ is changed to another variable $v$, then apply variable separable method.
- **Solution of Linear Differential Equation :** A differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P$ and $Q$ are functions of $x$ (or constants) can be solved as :
  1. Find Integrating Factor $(\text{I.F.})=e^{\int P\,dx}$
  2. The solution of the differential equation is $y(\text{I.F.})=\int Q\,(\text{I.F.})\,dx+C,$ where $C$ is constant of integration.

# Practice Time

## OBJECTIVE TYPE QUESTIONS

### Multiple Choice Questions (MCQs)

**1.** The order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x}$ is
(a) 3 (b) 2
(c) 1 (d) none of these

**2.** The order of the differential equation whose general solution is given by

$$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x + C_5}$$

where $C_1, C_2, C_3, C_4, C_5$ are arbitrary constants, is
(a) 5 (b) 4 (c) 3 (d) 2

**3.** The differential equation of all circles of radius $a$ is of order
(a) 2 (b) 3
(c) 4 (d) none of these

**4.** The differential equation of all parabolas whose axis of symmetry is along $x$-axis is of order
(a) 3 (b) 1
(c) 2 (d) none of these

**5.** The degree of differential equation of all curves having normal of constant length $c$, is
(a) 1 (b) 3
(c) 4 (d) none of these

**6.** The order and degree respectively of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2} \text{ is}$$

(a) 2, 4 (b) 3, $\frac{3}{2}$
(c) 3, not defined (d) 2, 2

**7.** The differential equation whose solution is $y = Ae^{3x} + Be^{-3x}$ is given by
(a) $y_2 - 3y_1 + 3y = 0$
(b) $xy_2 + 3y_1 - xy + x^2 + 3 = 0$
(c) $y_2 - 9y = 0$
(d) $(y_1)^3 - 3y(xy_2 - 3y) = 0$

**8.** The differential equation satisfied by

$y = \dfrac{A}{x} + B$ is ($A$, $B$ are parameters)

(a) $x^2y_1 = y$ (b) $xy_1 + 2y_2 = 0$
(c) $xy_2 + 2y_1 = 0$ (d) none of these

**9.** The differential equation whose solution represents the family $xy = Ae^{ax} + Be^{-ax}$ is

(a) $x\left(\dfrac{d^2y}{dx^2}\right)^2 + 2\dfrac{dy}{dx} = xy$

(b) $x\left(\dfrac{d^2y}{dx^2}\right)^2 + 2\dfrac{dy}{dx} = a^2xy$

(c) $x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} = a^2xy$

(d) none of these

**10.** The differential equation which represent the family of curves $y = ae^{bx}$, where $a$ and $b$ are arbitrary constants, is
(a) $y' = y^2$ (b) $y'' = y\,y'$
(c) $y\,y'' = y'$ (d) $y\,y'' = (y')^2$

**11.** The differential equation having solution as $y = 17e^x + ae^{-x}$ is
(a) $y'' - x = 0$ (b) $y'' - y = 0$
(c) $y' - y = 0$ (d) $y' - x = 0$.

**12.** The order of the differential equation of all tangent lines to the parabola $y = x^2$, is
(a) 1 (b) 2 (c) 3 (d) 4

**13.** The order of the differential equation whose general solution is given by $y = (A + B)\cos(x + C) + De^x$ is
(a) 4 (b) 3 (c) 2 (d) 1

**14.** The differential equation of all circles in the first quadrant which touch the coordinate axes is of order
(a) 1 (b) 2
(c) 3 (d) none of these

**15.** The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{2/3} + 4 - 3\frac{dy}{dx} = 0 \text{ is}$$

(a) 2 (b) 1
(c) 3 (d) none of these

**16.** The solution of the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$ satisfying $y(1) = 1$, is

(a) a hyperbola
(b) a circle
(c) $y^2 = x(1 + x) - 10$
(d) $(x - 2)^2 + (y - 3)^2 = 5xy$

**17.** Solution of the differential equation

$$\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots} = \frac{dx - dy}{dx + dy}, \text{ is}$$

(a) $2ye^{2x} = C \cdot e^{2x} + 1$
(b) $2ye^{2x} = C \cdot e^{2x} - 1$
(c) $ye^{2x} = C \cdot e^{2x} + 2$
(d) none of these

**18.** Given the differential equation

$\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$; $y(1) = \pi$.

Which of the following option is correct?

(a) Solution is $y^2 - \sin y = -2x^3 + C$
(b) Solution is $y^2 + \sin y = 2x^3 + C$
(c) $C = \pi + 2\sqrt{2}$
(d) $C = \pi^2 + 2$

**19.** For the differential equation $x\frac{dy}{dx} + 2y = xy\frac{dy}{dx}$,

(a) order is 1 and degree is 1
(b) solution is $\ln(yx^2) = C - y$
(c) order is 1 and degree is 2
(d) solution is $\ln(xy^2) = C + y$

**20.** The differential equation $\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{y}$ determines a family of circle with

(a) variable radii and fixed centre (0, 1)
(b) variable radii and fixed centre (0, –1)
(c) fixed radius 1 and variable centre on $x$-axis
(d) fixed radius 1 and variable centre on $y$-axis

**21.** If $y' = y + 1$, $y(0) = 1$, then $y(\ln 2) =$

(a) 1 (b) 2 (c) 3 (d) 4

**22.** The general solution of the differential equation $x(1 + y^2)\, dx + y\,(1 + x^2)\, dy = 0$ is

(a) $(1 + x^2)(1 + y^2) = 0$
(b) $(1 + x^2)(1 + y^2) = C$
(c) $(1 + x^2) = C(1 + y^2)$
(d) $(1 + y^4) = C(1 + x^2)$

**23.** If $\frac{dy}{dx} = y \sin 2x$, $y(0) = 1$, then solution is

(a) $y = e^{\sin^2 x}$
(b) $y = \sin^2 x$
(c) $y = \cos^2 x$
(d) $y = e^{\cos^2 x}$

**24.** The general solution of the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ is

(a) $x^3 - y^3 = c$
(b) $x^3 + y^3 = c$
(c) $x^2 + y^2 = c$
(d) $x^2 - y^2 = c$

**25.** The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(a) $y = \frac{k}{x}$
(b) $y = k \log x$
(c) $y = k\,x$
(d) $\log y = k\,x$

**26.** If $\frac{dy}{dx} = \frac{x + y}{x}$, $y(1) = 1$, then $y =$

(a) $x + \ln x$
(b) $x^2 + x \ln x$
(c) $xe^{x-1}$
(d) $x + x \ln x$

**27.** The order and degree respectively of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$, are

(a) 2 and not defined
(b) 2 and 2
(c) 2 and 3
(d) 3 and 3

**28.** Integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is

(a) $x$
(b) $\log x$
(c) $\frac{1}{x}$
(d) $-x$

**29.** The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is

(a) none
(b) one
(c) two
(d) infinite

**30.** Which of the following is a second order differential equation?

(a) $(y')^2 + x = y^2$
(b) $y'y'' + y = \sin x$
(c) $y''' + (y'')^2 + y = 0$
(d) $y' = y^2$

**31.** Integrating factor of the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is

(a) $\cos x$
(b) $\sec x$
(c) $e^{\cos x}$
(d) $e^{\sec x}$

**32.** The solution of the differential equation $\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$ is

(a) $y = \tan^{-1} x$
(b) $y - x = k(1 + xy)$
(c) $x = \tan^{-1} y$
(d) $\tan(xy) = k$

**33.** The integrating factor of the differential equation $\dfrac{dy}{dx}+y=\dfrac{1+y}{x}$ is

(a) $\dfrac{x}{e^x}$ (b) $\dfrac{e^x}{x}$ (c) $xe^x$ (d) $e^x$

**34.** The solution of $x\dfrac{dy}{dx}+y=e^x$ is

(a) $y=\dfrac{e^x}{x}+\dfrac{k}{x}$ (b) $y = xe^x + cx$

(c) $y = xe^x + k$ (d) $x=\dfrac{e^y}{y}+\dfrac{k}{y}$

**35.** Differential equation having solution $y = Ax + B^3$ is of order

(a) 3 (b) 2

(c) 1 (d) not defined

**36.** Integrating factor of the differential equation $\cos x\dfrac{dy}{dx}+y\sin x=1$ is

(a) $\cos x$ (b) $\tan x$ (c) $\sec x$ (d) $\sin x$

**37.** Solution of $\dfrac{dy}{dx}-y=1,\ y(0)=1$ is given by

(a) $xy = -e^x$ (b) $xy = -e^{-x}$

(c) $xy = -1$ (d) $y = 2e^x - 1$

**38.** Integrating factor of the differential equation $(1-x^2)\dfrac{dy}{dx}-xy=1$ is

(a) $-x$ (b) $\dfrac{x}{1+x^2}$

(c) $\sqrt{1-x^2}$ (d) $\dfrac{1}{2}\log(1-x^2)$

## Case Based MCQs

**Case I :** Read the following passage and answer the questions from 39 to 43.


A thermometer reading 80°F is taken outside. Five minutes later the thermometer reads 60°F. After another 5 minutes the thermometer reads 50°F. At any time $t$ the thermometer reading be $T$°F and the outside temperature be $S$°F.

**39.** If $\lambda$ is positive constant of proportionality, then $\dfrac{dT}{dt}$ is

(a) $\lambda(T-S)$ (b) $\lambda(T+S)$

(c) $\lambda TS$ (d) $-\lambda(T-S)$

**40.** The value of $T(5)$ is

(a) 30°F (b) 40°F (c) 50°F (d) 60°F

**41.** The value of $T(10)$ is

(a) 50°F (b) 60°F (c) 80°F (d) 90°F

**42.** Find the general solution of differential equation formed in given situation.

(a) $\log T = St + c$ (b) $\log(T-S) = -\lambda t + c$

(c) $\log S = tT + c$ (d) $\log(T+S) = \lambda t + c$

**43.** Find the value of constant of integration $c$ in the solution of differential equation formed in given situation.

(a) $\log(60-S)$ (b) $\log(80+S)$

(c) $\log(80-S)$ (d) $\log(60+S)$

**Case II :** Read the following passage and answer the questions from 44 to 48.

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let $P$ denotes the principal at any time $t$ and rate of interest be $r$ % per annum.

**44.** Find the value of $\dfrac{dP}{dt}$.

(a) $\dfrac{Pr}{1000}$ (b) $\dfrac{Pr}{100}$ (c) $\dfrac{Pr}{10}$ (d) $Pr$

**45.** If $P_0$ be the initial principal, then find the solution of differential equation formed in given situation.

(a) $\log\left(\dfrac{P}{P_0}\right)=\dfrac{rt}{100}$ (b) $\log\left(\dfrac{P}{P_0}\right)=\dfrac{rt}{10}$

(c) $\log\left(\dfrac{P}{P_0}\right)=rt$ (d) $\log\left(\dfrac{P}{P_0}\right)=100rt$

**46.** If the interest is compounded continuously at 5% per annum, then in how many years will ₹ 100 double itself ? ($\log_e 2 = 0.6931$)

(a) 12.728 years (b) 14.789 years

(c) 13.862 years (d) 15.872 years

**47.** At what interest rate will ₹ 100 double itself in 10 years? ($\log_e 2 = 0.6931$).
(a) 9.66% (b) 8.239% (c) 7.341% (d) 6.931%

**48.** How much will ₹ 1000 be worth at 5% interest after 10 years? ($e^{0.5} = 1.648$).
(a) ₹ 1648 (b) ₹ 1500 (c) ₹ 1664 (d) ₹ 1572

**Case III :** Read the following passage and answer the questions from 49 to 53.

In a college hostel accommodating 1000 students, one of the hostellers came in carrying Corona virus, and the hostel was isolated. The rate at which the virus spreads is assumed to be proportional to the product of the number of infected students and remaining students. There are 50 infected students after 4 days.


**49.** If $n(t)$ denote the number of students infected by Corona virus at any time $t$, then maximum value of $n(t)$ is
(a) 50 (b) 100 (c) 500 (d) 1000

**50.** $\frac{dn}{dt}$ is proportional to
(a) $n(1000 - n)$ (b) $n(1000 + n)$
(c) $n(100 - n)$ (d) $n(100 + n)$

**51.** The value of $n(4)$ is
(a) 1 (b) 50
(c) 100 (d) 1000

**52.** The most general solution of differential equation formed in given situation is
(a) $\frac{1}{1000}\log\left(\frac{1000-n}{n}\right) = \lambda t + c$
(b) $\log\left(\frac{n}{100-n}\right) = \lambda t + c$
(c) $\frac{1}{1000}\log\left(\frac{n}{1000-n}\right) = \lambda t + c$
(d) None of these

**53.** The value of $n$ at any time is given by
(a) $n(t) = \frac{1000}{1+999e^{-0.9906t}}$
(b) $n(t) = \frac{1000}{1-999e^{-0.9906t}}$
(c) $n(t) = \frac{100}{1-999e^{-0.996t}}$
(d) $n(t) = \frac{100}{999+e^{1000t}}$

## Assertion & Reasoning Based MCQs

**Directions (Q.-54 to 60) :** In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :
*(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.*
*(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.*
*(c) Assertion is correct statement but Reason is wrong statement.*
*(d) Assertion is wrong statement but Reason is correct statement.*

**54. Assertion :** The differential equation of all circles in a plane must be of order 3.
**Reason :** If three points are non-collinear, then only one circle passes through these points.

**55. Assertion :** Order of the differential equation whose solution is
$y = c_1 e^{x + c_2} + c_3 e^{x + c_4}$ is 4.
**Reason :** Order of the differential equation is equal to the number of independent arbitrary constants mentioned in the solution of the differential equation.

**56. Assertion :** $y = a \sin x + b \cos x$ is a general solution of $y'' + y = 0$.
**Reason :** $y = a \sin x + b \cos x$ is a trigonometric function.

**57. Assertion :** The elimination of four arbitrary constants in $y = (c_1 + c_2 + c_3 e^{c_4})x$ results into a differential equation of the first order $x\frac{dy}{dx} = y$.

**Reason :** Elimination of $n$ arbitrary constants requires in general, a differential equation of the $n^{th}$ order.

**58. Assertion :** '$x$' is not an integrating factor for the differential equation $x\dfrac{dy}{dx} + 2y = e^x$.

**Reason :** $x\left(x\dfrac{dy}{dx}+2y\right)=\dfrac{d}{dx}(x^2y)$.

**59. Assertion :** $x\sin x\dfrac{dy}{dx}+(x+x\cos x+\sin x)y=\sin x$,

$y\left(\dfrac{\pi}{2}\right) = 1-\dfrac{2}{\pi} \Rightarrow \lim_{x\to 0} y(x) = \dfrac{1}{3}$.

**Reason :** The differential equation is linear with integrating factor $x(1 - \cos x)$

**60. Assertion :** If $\dfrac{dy}{dx} + xy = x^3y^3$, $x > 0$, $y \geq 0$ and $y(0) = 1$, then $y(1) = \dfrac{1}{\sqrt{2}}$.

**Reason :** The differential equation is linear with integrating factor $e^x$.

# SUBJECTIVE TYPE QUESTIONS

## Very Short Answer Type Questions (VSA)

**1.** Find the order and the degree of the differential equation $x^2\dfrac{d^2y}{dx^2}=\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^4$.

**2.** Write the sum of the order and degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2+\left(\dfrac{dy}{dx}\right)^3+x^4=0$.

**3.** Write the degree of differential equation $x^3\left(\dfrac{d^2y}{dx^2}\right)^2+x\left(\dfrac{dy}{dx}\right)^4=0$.

**4.** Find the differential equation whose solution is $v=\dfrac{A}{r}+B$, where $A$ and $B$ are arbitrary constants.

**5.** Find the differential equation whose solution is $y = mx$, where $m$ is an arbitrary constant.

**6.** Find the integrating factor of the differential equation $(y+3x^2)\dfrac{dx}{dy}=x$.

**7.** Find the integrating factor of the differential equation $\left(\dfrac{e^{-2\sqrt{x}}}{\sqrt{x}}-\dfrac{y}{\sqrt{x}}\right)\dfrac{dx}{dy}=1$.

**8.** Write the integrating factor of the differential equation $(1+x^2)+(2yx-\cot x)\dfrac{dx}{dy}=0$.

**9.** Write the solution of the differential equation $\dfrac{dy}{dx}=2^{-y}$.

**10.** Find the solution of the differential equation $\dfrac{dy}{dx}=x^3e^{-2y}$.

## Short Answer Type Questions (SA-I)

**11.** Find the differential equation whose solution is $y = e^{2x}(a + bx)$, where '$a$' and '$b$' are arbitrary constants.

**12.** Find the differential equation whose solution is $y = ae^{bx+5}$, where $a$ and $b$ are arbitrary constants.

**13.** Solve the differential equation $\dfrac{dy}{dx}+y=\cos x-\sin x$.

**14.** Find the general solution of the differential equation $xe^{y/x}\,dy = (ye^{y/x} + x^2)dx$, $x \neq 0$

**15.** Solve the differential equation $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

**16.** Find the particular solution of the differential equation $(1 - y^2)(1 + \log x)dx + 2xy\,dy = 0$, given that $y = 0$ when $x = 1$.

**17.** Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$, given that $y = \frac{\pi}{2}$, when $x = 1$.

**18.** Find the particular solution of the differential equation $x(1+y^2)\,dx - y(1+x^2)dy = 0$, given that $y = 1$ when $x = 0$.

**19.** Find the particular solution of the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$; $y = -1$ when $x = 1$.

**20.** Find the particular solution of the differential equation $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$; $y = 0$ when $x = 0$.

## Short Answer Type Questions (SA-II)

**21.** Find the differential equation whose solution is $(x + a)^2 + (y - a)^2 = a^2$, where $a$ is constant.

**22.** Find the differential equation whose solution is $x^2 = 4ay$, where $a$ is constant.

**23.** Find the general solution of the differential equation $xe^x\,dy = (x^3 + 2y\,e^x)dx$.

**24.** Find the particular solution of the differential equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$, given that $y = \frac{\pi}{4}$ at $x = 1$.

**25.** Solve the differential equation $xdy - ydx = \sqrt{x^2+y^2}\,dx$, given that $y = 0$ when $x = 1$.

**26.** Solve the differential equation $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, given $y(0) = 0$.

**27.** Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

**28.** Find the particular solution of the differential equation $e^x \tan y\,dx + (2 - e^x)\sec^2 y\,dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.

**29.** Solve the following differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$

**30.** Solve the differential equation $(x^2-1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}, |x| \neq 1$

**31.** Find the particular solution of the differential equation $\frac{dy}{dx} = 1 + x + y + xy$, given that $y = 0$ when $x = 1$

**32.** Solve the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$, given that $y = 0$ when $x = \frac{\pi}{2}$.

**33.** If $y(x)$ is a solution of the differential equation $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of $y\left(\frac{\pi}{2}\right)$.

**34.** Find the particular solution of the following differential equation $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$, given that when $x = 2$, $y = \pi$.

**35.** Show that the differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ is homogeneous and also solve it.

## Long Answer Type Questions (LA)

**36.** Find the particular solution of the differential equation $(x-y)\frac{dy}{dx} = (x+2y)$, given that $y = 0$ when $x = 1$.

**37.** Find the particular solution of the differential equation $(\tan^{-1}x - y)\,dx = (1+x^2)dy$, given that $y = 1$ when $x = 0$.

**38.** Solve the following differential equation :

$$\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx + xy\,dy = 0$$

**39.** Find the particular solution of the differential equation $(x - y)\,\frac{dy}{dx} = x + 2y$, given that when $x = 1$, $y = 0$.

**40.** Prove that $x^2 - y^2 = C(x^2 + y^2)^2$ is the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)\,dy$, where $C$ is a parameter.

# ANSWERS

## OBJECTIVE TYPE QUESTIONS

**1.** **(a) :** $y = a\cos x + b\sin x + ce^{-x}$ is a three parameter family of curve, it is a third order differential equation.

**2.** **(c) :** The given equation can be rewritten as

$$y = A\cos(x + C_3) - Be^x$$

where $A = C_1 + C_2$ and $B = C_4 e^{C_5}$

So, there are three independent variables, $(A, B, C_3)$. Hence, the differential equation is of order 3.

**3.** **(a) :** The equation of the family of circles of radius $a$ is $(x - h)^2 + (y - k)^2 = a^2$ which is a two parameter family of curve. So, its differential equation is of order two.

**4.** **(c) :** The general form of the equation of parabola whose symmetry is along $x$-axis is given by $x = ay^2 + b$, which is the solution of the differential equation of all such type of parabolas which contains two arbitrary constants. So, the order of the differential equation is 2.

**5.** **(d) :** We have, $y\sqrt{1+\left(\frac{dy}{dx}\right)^2} = c$

$$\Rightarrow y^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\} = c^2 \Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + y^2 = c^2$$

Clearly, it is a differential equation of degree 2.

**6.** **(d) :** The given differential equation can be written as

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Clearly, order and degree of given differential equation are 2, 2 respectively.

**7.** **(c) :** We have, $y = Ae^{3x} + Be^{-3x}$

Differentiating w.r.t. $x$, we get

$y_1 = 3Ae^{3x} - 3Be^{-3x}$

Again differentiating w.r.t. $x$, we get

$y_2 = 9Ae^{3x} + 9Be^{-3x} = 9(Ae^{3x} + Be^{-3x}) = 9y$

$\Rightarrow\ y_2 - 9y = 0$

**8.** **(c) :** Given relation is $y = \frac{A}{x} + B$

Differentiating w.r.t. $x$, we get

$y_1 = -\frac{A}{x^2} \Rightarrow x^2 y_1 = -A$

Again differentiating w.r.t. $x$, we get

$x^2y_2 + y_1 2x = 0 \ \Rightarrow\ xy_2 + 2y_1 = 0$

**9.** **(c) :** Given $xy = Ae^{ax} + Be^{-ax}$ ...(i)

Differentiating (i) w.r.t. $x$, we get

$$y + x\frac{dy}{dx} = a(Ae^{ax} - Be^{-ax})$$

Differentiating again w.r.t. $x$, we get

$$\frac{dy}{dx} + x\frac{d^2y}{dx^2} + \frac{dy}{dx} = a^2(Ae^{ax} + Be^{-ax})$$

$$\Rightarrow x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = a^2(xy) \quad \text{(Using (i))}$$

**10.** **(d) :** We have, $y = ae^{bx} \Rightarrow \ln y = \ln a + bx$

Differentiating w.r.t. $x$, we get $\frac{1}{y}y' = b$

Again differentiating w.r.t. $x$, we get

$$\frac{y''}{y} - \frac{1}{y^2}(y')^2 = 0 \quad \Rightarrow \quad y\,y'' = (y')^2$$

**11.** **(b) :** We have $y = 17e^x + ae^{-x} \Rightarrow y' = 17e^x - ae^{-x}$

$\Rightarrow y'' = 17e^x + ae^{-x} \Rightarrow y'' = y \Rightarrow y'' - y = 0$

**12.** **(a) :** The parametric form of the given equation is $x = t$, $y = t^2$.

The equation of any tangent at $t$ is $2xt = y + t^2$

On differentiating, we get $2t = y_1$

On putting this value in the above equation, we get

$$xy_1 = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1^2$$

The order of this equation is 1.

**13.** **(b) :** Given $y = (A + B)\cos(x + C) + De^x$

This can be rewritten as,

$y = k\cos(x + C) + De^x$

Now order of a differential is same as the number of arbitrary unknowns present in the solution.

Hence, order of the given differential equation is 3.

**14.** **(a) :** The equation of the family of circles which touch both the axes is $(x - a)^2 + (y - a)^2 = a^2$, where $a$ is a parameter. This is one parameter family of curve. So, the order of differential equation is one.

**15.** **(a) :** $\left(\frac{d^2y}{dx^2}\right)^{2/3} = 3\frac{dy}{dx} - 4 \Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left(3\frac{dy}{dx} - 4\right)^3$

$\therefore$ Degree of the differential equation is 2.

**16.** **(a) :** Given, $\frac{dy}{dx} = \frac{x^2 + y^2 + 1}{2xy}$

$\Rightarrow 2xydy = (x^2 + y^2 + 1)dx \Rightarrow 2xydy - y^2dx = (x^2 + 1)dx$

$\Rightarrow xd(y^2) - y^2dx = (x^2 + 1)dx$

$$\Rightarrow \frac{xd(y^2) - y^2dx}{x^2} = \left(1 + \frac{1}{x^2}\right)dx \Rightarrow d\left(\frac{y^2}{x}\right) = d\left(x - \frac{1}{x}\right)$$

Integrating both sides, we get

$$\frac{y^2}{x} = x - \frac{1}{x} + C$$

$$\Rightarrow y^2 = x^2 - 1 + Cx \Rightarrow y^2 = \left(x + \frac{C}{2}\right)^2 - 1 - \frac{C^2}{4}$$

Clearly, it represents a hyperbola.

**17.** **(b) :** Given equation can be rewritten as

$$\frac{\frac{1}{2}(e^x - e^{-x})}{\frac{1}{2}(e^x + e^{-x})} = \frac{dx - dy}{dx + dy}$$

Applying componendo and dividendo, we get

$\frac{dy}{dx} = \frac{e^{-x}}{e^x} \Rightarrow dy = e^{-2x}dx \Rightarrow 2y = -e^{-2x} + C$ (Integrating both sides)

$\Rightarrow \quad 2ye^{2x} = C \cdot e^{2x} - 1$

**18.** **(b) :** We have, $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$

$\Rightarrow \quad \int (2y + \cos y)\, dy = \int 6x^2 dx + C$

$\Rightarrow \quad y^2 + \sin y = 2x^3 + C$

$\because \quad y(1) = \pi \Rightarrow C = \pi^2 - 2$

$\therefore$ Solution is $y^2 + \sin y = 2x^3 + \pi^2 - 2$

$\Rightarrow \quad y^2 + \sin y = 2x^3 + C$, where $C = \pi^2 - 2$

**19.** **(a) :** Given, $x\frac{dy}{dx}(1 - y) + 2y = 0$

$\Rightarrow \left(\frac{1-y}{y}\right)dy + 2\frac{dx}{x} = 0 \Rightarrow \frac{1}{y}dy - dy + 2\frac{dx}{x} = 0$

On integrating, we get

$\Rightarrow \quad \ln y - y + 2\ln x = C \Rightarrow \ln(yx^2) = C + y$

**20.** **(c) :** We have, $\frac{ydy}{\sqrt{1-y^2}} = dx$

On integration, we get $-\sqrt{1-y^2} = x + c$

$\Rightarrow \quad 1 - y^2 = (x + c)^2 \Rightarrow (x + c)^2 + y^2 = 1$

which is a circle of radius 1 and centre on the $x$-axis.

**21.** **(c) :** $y' = y + 1 \Rightarrow \frac{dy}{y+1} = dx$

$\Rightarrow \quad \ln(y + 1) = x + C$ (Integrating both sides)

Now, $y(0) = 1 \Rightarrow C = \ln 2$

$\therefore \ln\left(\frac{y+1}{2}\right) = x \Rightarrow y + 1 = 2e^x \Rightarrow y = 2e^x - 1$

So, $y(\ln 2) = 2e^{\ln 2} - 1 = 4 - 1 = 3$.

**22.** **(b) :** Given differential equation is

$x(1 + y^2)dx + y(1 + x^2)dy = 0$

$\Rightarrow \left(\frac{x}{1+x^2}\right)dx + \left(\frac{y}{1+y^2}\right)dy = 0$

On integrating, we get

$$\frac{1}{2}\log(1+x^2) + \frac{1}{2}\log(1+y^2) = k$$

$\Rightarrow \quad \log(1 + x^2)(1 + y^2) = 2k \Rightarrow (1 + x^2)(1 + y^2) = e^{2k} = C$

**23.** **(a) :** We have $\frac{dy}{dx} = y \sin 2x$

$\Rightarrow \quad \frac{dy}{y} = \sin 2x\, dx \Rightarrow \log y = -\frac{\cos 2x}{2} + c$

Since $y(0) = 1 \Rightarrow x = 0, y = 1 \Rightarrow c = 1/2$

Now, $\log y = \frac{1}{2}(1 - \cos 2x) \Rightarrow \log y = \sin^2 x \Rightarrow y = e^{\sin^2 x}$

**24.** **(a) :** $\frac{dy}{dx} = \frac{x^2}{y^2} \Rightarrow y^2 dy = x^2 dx$

$\Rightarrow \int y^2 dy = \int x^2 dx + C \Rightarrow \frac{y^3}{3} = \frac{x^3}{3} + C$

$\Rightarrow \quad x^3 - y^3 = -3C = c$ (say).

**25.** **(c) :** $\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dx}{x} = \frac{dy}{y}$

$\Rightarrow \log x = \log y + \log c \Rightarrow x = yc \Rightarrow y = \frac{1}{c}x \Rightarrow y = kx$.

**26.** **(d) :** It is a homogeneous equation.

Substitute $y = vx \Rightarrow \frac{dy}{dx} = \frac{xdv}{dx} + v$

Now, given equation becomes

$\frac{xdv}{dx} + v = 1 + v \Rightarrow dv = \frac{dx}{x}$

On integrating both sides, we get

$v = \ln x + c \Rightarrow \frac{y}{x} = \ln x + c$

$\because \quad y(1) = 1 \Rightarrow x = 1, y = 1 \Rightarrow c = 1 \quad \therefore \quad y = x + x\ln x$.

**27.** **(a) :** $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{4}} + x^{\frac{1}{5}} = 0$

Clearly, order of given differential equation is 2 and degree is not defined.

**28.** **(c) :** $\frac{xdy}{dx} - y = x^4 - 3x$

$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = \frac{x^4 - 3x}{x} \Rightarrow \frac{dy}{dx} - \frac{1}{x}\cdot y = x^3 - 3$

$\therefore \quad$ I.F. $= e^{-\int \frac{1}{x}dx} = e^{-\log x} = e^{\log(x)^{-1}} = x^{-1} = \frac{1}{x}$

**29.** **(b) :** $\frac{dy}{dx} = \frac{y+1}{x-1} \Rightarrow \frac{dy}{y+1} = \frac{dx}{x-1}$

On integrating both sides, we get

$\log(y + 1) + \log c = \log(x - 1) \Rightarrow (y + 1)c = (x - 1)$

Now, $y(1) = 2 \Rightarrow 3c = 0 \Rightarrow c = 0$

$\therefore \quad x - 1 = 0 \Rightarrow x = 1$

Hence, only one solution exists.

**30.** **(b) :** (a) $\left(\frac{dy}{dx}\right)^2 + x = y^2$ ; order = 1

(b) $\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right) + y = \sin x$ ; order = 2

(c) $\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^2 + y = 0$ ; order = 3

(d) $\frac{dy}{dx} = y^2$ ; order = 1

**31.** **(b) :** $\frac{dy}{dx} + y\tan x - \sec x = 0$

$\therefore \quad$ I.F. $= e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$

**32. (b) :** $\frac{dy}{dx}=\frac{1+y^2}{1+x^2} \Rightarrow \frac{dy}{1+y^2}=\frac{dx}{1+x^2}$

On integrating both sides, we get

$\tan^{-1}y = \tan^{-1}x + c \Rightarrow \tan^{-1}y - \tan^{-1}x = c$

$\Rightarrow \quad \tan^{-1}\left(\frac{y-x}{1+xy}\right) = c \Rightarrow \frac{y-x}{1+xy} = \tan c = k$(say)

$\Rightarrow \quad y - x = k(1 + xy)$

**33. (b) :** $\frac{dy}{dx}+y=\frac{1+y}{x}$

$\Rightarrow \quad \frac{dy}{dx}+y=\frac{1}{x}+\frac{y}{x} \Rightarrow \frac{dy}{dx}+y-\frac{y}{x}=\frac{1}{x}$

$\Rightarrow \quad \frac{dy}{dx}+\left(1-\frac{1}{x}\right)y=\frac{1}{x}$

$\therefore \quad$ I.F. $= e^{\int\left(1-\frac{1}{x}\right)dx} = e^{x-\log x} = e^x e^{-\log x} = e^x e^{\log(x)^{-1}}$

$= e^x x^{-1} = \frac{e^x}{x}$

**34. (a) :** $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx}+\frac{1}{x}y=\frac{e^x}{x}$

It is a linear differential equation with

I.F. $= e^{\int\frac{1}{x}dx} = e^{\log x} = x$

Now, solution is $y\cdot x = \int\frac{e^x}{x}\cdot x\,dx + k$

$\Rightarrow \quad yx = e^x + k \quad \Rightarrow \quad y=\frac{e^x}{x}+\frac{k}{x}$

**35. (b) :** $y = Ax + B^3$

Differentiating w.r.t. $x$, we get $\frac{dy}{dx} = A$

Again differentiating w.r.t. $x$, we get $\frac{d^2y}{dx^2}=0$

which is a differential equation of order 2.

**36. (c) :** $\cos x\frac{dy}{dx} + y\sin x = 1$

$\Rightarrow \quad \frac{dy}{dx}+\frac{\sin x}{\cos x}\cdot y=\frac{1}{\cos x} \Rightarrow \frac{dy}{dx} + \tan x\cdot y = \sec x$

$\therefore \quad$ I.F. $= e^{\int\tan x dx} = e^{\log(\sec x)} = \sec x$

**37. (d) :** $\frac{dy}{dx} - y = 1 \Rightarrow \frac{dy}{dx} = 1 + y \Rightarrow \frac{dy}{1+y} = dx$

On integrating, we get

$\log(1 + y) = x + c$

Now, $y(0) = 1 \Rightarrow \log 2 = c$

$\therefore \quad \log(1 + y) = x + \log 2$

$\Rightarrow \quad \frac{1+y}{2} = e^x \Rightarrow 1+ y = 2e^x \Rightarrow y = 2e^x - 1$

**38. (c) :** $(1 - x^2)\frac{dy}{dx} - xy = 1 \Rightarrow \frac{dy}{dx}-\frac{x}{1-x^2}\cdot y=\frac{1}{1-x^2}$

$\therefore \quad$ I.F. $= e^{-\int\frac{x}{1-x^2}dx} = e^{\frac{1}{2}\int\frac{-2x}{1-x^2}dx}$

$= e^{\frac{1}{2}\log(1-x^2)} = e^{\log(1-x^2)^{\frac{1}{2}}} = \sqrt{1-x^2}$

**39. (d) :** Given, at any time $t$ the thermometer reading be $T$°F and the outside temperature be $S$°F. Then, by Newton's law of cooling, we have

$\frac{dT}{dt}\propto(T-S) \Rightarrow \frac{dT}{dt}=-\lambda(T-S)$

**40. (d) :** Since, after 5 minutes, thermometer reads 60°F

$\therefore \quad$ Value of $T(5)$ = 60°F

**41. (a):** Clearly from given information, value of $T(10)$ is 50°F.

**42. (b):** We have, $\frac{dT}{dt}=-\lambda(T-S)$

$\Rightarrow \quad \frac{dT}{T-S}=-\lambda dt \Rightarrow \int\frac{1}{T-S}dT=-\lambda\int dt$

$\Rightarrow \quad \log(T - S) = -\lambda t + c$

**43. (c):** Since, at $t = 0$, $T$ = 80°F

$\therefore \quad \log(80 - S) = 0 + c \Rightarrow c = \log(80 - S)$

**44. (b) :** Here, $P$ denotes the principal at any time $t$ and the rate of interest be $r$% per annum compounded continuously, then according to the law given in the problem, we get $\frac{dP}{dt}=\frac{Pr}{100}$

**45. (a):** We have, $\frac{dP}{dt}=\frac{Pr}{100}$

$\Rightarrow \quad \frac{dP}{P}=\frac{r}{100}dt \Rightarrow \int\frac{1}{P}dP=\frac{r}{100}\int dt$

$\Rightarrow \quad \log P=\frac{rt}{100}+C \quad$ ...(1)

At $t = 0$, $P = P_0$.

$\therefore \quad C = \log P_0$

So, $\log P=\frac{rt}{100}+\log P_0$

$\Rightarrow \quad \log\left(\frac{P}{P_0}\right)=\frac{rt}{100} \quad$ ...(2)

**46. (c):** We have, $r = 5$, $P_0$ = ₹ 100 and $P = 2P_0$ = ₹ 200

Substituting these values in (2), we get

$\log 2=\frac{5}{100}t$

$\Rightarrow \quad t = 20\log_e 2 = 20 \times 0.6931$ years = 13.862 years

**47. (d) :** We have,

$P_0$ = ₹ 100, $P = 2P_0$ = ₹ 200 and $t$ = 10 years

Substituting these values in (2), we get

$\log 2=\frac{10r}{100} \Rightarrow r = 10\log 2 = 10 \times 0.6931 = 6.931$ %

**48. (a):** We have

$P_0$ = ₹ 1000, $r$ = 5 and $t$ = 10

Substituting these values in (2), we get

$$\log\left(\frac{P}{1000}\right)=\frac{5\times10}{100}\Rightarrow\log\left(\frac{P}{1000}\right)=0.5\Rightarrow\frac{P}{1000}=e^{0.5}$$

$\Rightarrow$ $P = 1000 \times 1.648$ = ₹ 1648

**49. (d):** Since, maximum number of students in hostel is 1000.

$\therefore$ Maximum value of $n(t)$ is 1000.

**50. (a):** Clearly, according to given information,

$\frac{dn}{dt}=\lambda n(1000-n)$, where $\lambda$ is constant of proportionality.

**51. (b):** Since, 50 students are infected after 4 days.

$\therefore$ $n(4) = 50$.

**52. (c):** We have, $\frac{dn}{dt}=\lambda n(100-n)$

$$\Rightarrow \int\frac{dn}{n(1000-n)}=\lambda\int dt$$

$$\Rightarrow \frac{1}{1000}\int\left(\frac{1}{1000-n}+\frac{1}{n}\right)dn=\lambda\int dt$$

$$\Rightarrow \frac{1}{1000}\left[\frac{\log(1000-n)}{-1}+\log n\right]=\lambda t+c$$

$$\Rightarrow \frac{1}{1000}\log\left(\frac{n}{1000-n}\right)=\lambda t+c$$

**53. (a):** When, $t = 0$, $n = 1$

This condition is satisfied by option (a) only.

**54. (b):** Let $x^2 + y^2 + 2gx + 2fy + c = 0$

Here, in this equation, there are three constants.

$\therefore$ Order = 3

Reason is also correct.

**55. (d):** $\because$ $y = (c_1e^{c_2} + c_3e^{c_4})\, e^x = ce^x$ ...(i)

$\therefore$ $\frac{dy}{dx}=ce^x\Rightarrow\frac{dy}{dx}=y$ (Using (i))

$\therefore$ Order is 1.

**56. (b):** $\because$ $y = a\sin x + b\cos x$ ...(i)

$\therefore$ $y' = a\cos x - b\sin x$

$\Rightarrow$ $y'' = -a\sin x - b\cos x = -y$ [Using (i)]

$\Rightarrow$ $y'' + y = 0$

**57. (a) :** Let $c_1 + c_2 + c_3e^{c_4} = A$ (constant)

Then, $y = Ax$ ...(i)

$\Rightarrow$ $\frac{dy}{dx}=A \Rightarrow \frac{dy}{dx}=\frac{y}{x}$ [Using(i)]

$\Rightarrow$ $x\frac{dy}{dx}=y$

**58. (b):** $\frac{dy}{dx}+\frac{2}{x}y=\frac{e^x}{x}$

I.F. = $e^{\int\frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

$\Rightarrow$ Assertion is correct.

Now, $\frac{d}{dx}(x^2 y) = x^2\frac{dy}{dx} + y\cdot 2x = x\left(x\frac{dy}{dx}+2y\right)$

$\Rightarrow$ Reason is correct.

**59. (a) :** $\frac{dy}{dx}+\left(\frac{1}{\sin x}+\cot x+\frac{1}{x}\right)^y=\frac{1}{x}$

I.F.$=\exp\int\left(\frac{1}{\sin x}+\cot x+\frac{1}{x}\right)dx = \exp\ln\left(x\tan\frac{x}{2}\sin x\right)$

$= x\tan\frac{x}{2}\times2\sin\frac{x}{2}\cos\frac{x}{2} = x(1-\cos x)$

$\therefore$ Solution is, $yx(1-\cos x)=\int\frac{1}{x}\cdot x(1-\cos x)dx$

$= x - \sin x + c$

$y\left(\frac{\pi}{2}\right) = 1-\frac{2}{\pi} \Rightarrow c = 0$

$\therefore$ $y=\frac{x-\sin x}{x(1-\cos x)}$

$$\Rightarrow y=\frac{x-\left(x-\frac{x^3}{6}...\right)}{x\left(1-\left(1-\frac{x^2}{2}...\right)\right)}=\frac{\frac{x^2}{6}\left(1-\frac{x^2}{20}+....\right)}{\frac{x^2}{2}\left(1-\frac{x^2}{12}+....\right)}$$

As $x\to0$, $y\to\frac{1}{3}$

**60. (c) :** $\frac{1}{y^3}\frac{dy}{dx}+\frac{x}{y^2}=x^3$

Put $\frac{1}{y^2}=z \Rightarrow -\frac{2}{y^3}dy=dz$

$\therefore$ $\frac{dz}{dx}-2xz=-2x^3$,

which is a linear differential equation with I.F. = $e^{-x^2}$

$\therefore$ Solution, $ze^{-x^2}=-\int e^{-x^2}2x^3dx$

$\Rightarrow$ $ze^{-x^2}=(x^2+1)e^{-x^2}+C \Rightarrow z=x^2+1+Ce^{x^2}$

$\therefore$ $\frac{1}{y^2}=x^2+1+Ce^{x^2}$

$\because$ $y(0) = 1 \Rightarrow C = 0$

$\therefore$ $y^2=\frac{1}{x^2+1}\Rightarrow y=\frac{1}{\sqrt{x^2+1}}\Rightarrow y(1)=\frac{1}{\sqrt{2}}$.

## SUBJECTIVE TYPE QUESTIONS

**1.** The given differential equation is

$$x^2\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^4$$

$\therefore$ Its order is 2 and degree is 1.

**2.** Order = 2, Degree = 2.

$\therefore$ Required Sum = 2 + 2 = 4

**3.** Degree of the given differential equation is 2.

**4.** $v=\frac{A}{r}+B \Rightarrow \frac{dv}{dr}=-\frac{A}{r^2} \Rightarrow \frac{d^2v}{dr^2}=\frac{2A}{r^3}$

Now, $\frac{d^2v}{dr^2} \div \frac{dv}{dr} = \frac{2A}{r^3} \div \left(\frac{-A}{r^2}\right)$

$\Rightarrow \frac{d^2v}{dr^2} \div \frac{dv}{dr} = \frac{-2}{r} \Rightarrow \frac{d^2v}{dr^2} = \frac{-2}{r} \cdot \frac{dv}{dr}$

$\Rightarrow \frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} = 0$ is the required differential equation.

**5.** Here, $y = mx$ ...(i)

Differentiating (i) w.r.t. $x$, we get

$\frac{dy}{dx} = m$ ...(ii)

Eliminating $m$ from (i) and (ii), we get

$y = x \cdot \frac{dy}{dx} \Rightarrow x\frac{dy}{dx} - y = 0,$ is the required differential equation.

**6.** **(c) :** We have, $(y + 3x^2)\frac{dx}{dy} = x$

$\Rightarrow \frac{y + 3x^2}{x} = \frac{dy}{dx} \Rightarrow \frac{dy}{dx} - \frac{y}{x} = 3x$

This is a linear differential equation.

$\therefore$ I.F. $= e^{-\int \frac{dx}{x}} = e^{-\log x} = e^{\log(x)^{-1}} = \frac{1}{x}$

**7.** We have, $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

or $\frac{dy}{dx} + \frac{1}{\sqrt{x}}y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$

I.F. $= e^{\int \frac{1}{\sqrt{x}}dx} = e^{2\sqrt{x}}$

**8.** The given differential equation is

$(1 + x^2) + (2xy - \cot x)\frac{dx}{dy} = 0$

$\Rightarrow (1 + x^2)\frac{dy}{dx} + 2xy - \cot x = 0$

$\Rightarrow \frac{dy}{dx} + \frac{2x}{1 + x^2} \cdot y = \frac{\cot x}{1 + x^2}$

$\therefore$ I.F. $= e^{\int \frac{2x}{1+x^2}dy} = e^{\log(1+x^2)} = 1 + x^2$.

**9.** We have, $\frac{dy}{dx} = 2^{-y}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2^y} \Rightarrow 2^y\, dy = dx$ ...(i)

Integrating both sides of (i), we get

$\frac{2^y}{\log 2} = x + C \Rightarrow 2^y = (C + x) \log 2$

Taking log on both sides to the base 2, we get

$\log_2 2^y = \log_2 [(C + x) \log 2]$

$\Rightarrow y = \log_2 [(C + x) \log 2]$,

which is the required solution.

**10.** We have, $\frac{dy}{dx} = x^3 e^{-2y} \Rightarrow e^{2y}\, dy = x^3 dx$

On integrating, we get $\frac{e^{2y}}{2} = \frac{x^4}{4} + C'$

$\Rightarrow 2e^{2y} = x^4 + C$, where $C = 4C'$

**11.** We have $y = e^{2x}(a + bx)$ ...(i)

On differentiating (i) w.r.t. $x$, we get

$\frac{dy}{dx} = 2e^{2x}(a + bx) + be^{2x} = 2y + be^{2x}$ ...(ii)

Again differentiating (ii) w.r.t. $x$, we get

$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} + 2be^{2x} = 2\frac{dy}{dx} + 2\left(\frac{dy}{dx} - 2y\right)$ [Using (ii)]

$= 4\frac{dy}{dx} - 4y$

Hence, the required differential equation is

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0.$

**12.** We have, $y = a\, e^{bx+5}$ ...(i)

Differentiating (i) w.r.t. $x$, we get

$\frac{dy}{dx} = a\,e^{bx+5} \cdot b \Rightarrow e^{bx+5} = \frac{1}{ab}\frac{dy}{dx}$ ...(ii)

Differentiating (ii) w.r.t. $x$, we get

$b \cdot e^{bx+5} = \frac{1}{ab}\frac{d^2y}{dx^2} \Rightarrow e^{bx+5} = \frac{1}{ab^2}\frac{d^2y}{dx^2}$ ...(iii)

From (ii) & (iii), we have ...(iv)

$\frac{1}{ab}\frac{dy}{dx} = \frac{1}{ab^2}\frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dx^2} = b\frac{dy}{dx}$

From (i) and (ii), we have, $\frac{1}{y}\frac{dy}{dx} = b$ ...(v)

From (iv) and (v), we get $\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$ which is the required differential equation.

**13.** We have, $\frac{dy}{dx} + y = \cos x - \sin x$, which is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = 1$, $Q = \cos x - \sin x$

$\therefore$ I.F. $= e^{\int dx} = e^x$

The solution of the given differential equation is

$ye^x = \int e^x(\cos x - \sin x)dx + C$

$\Rightarrow ye^x = e^x \cos x + \int e^x \sin x - \int e^x \sin x + C$

$\Rightarrow ye^x = e^x\cos x + C \Rightarrow y = \cos x + Ce^{-x}$

**14.** $xe^{y/x}dy = (ye^{y/x} + x^2)dx$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \frac{x}{e^{y/x}}$

Putting $\frac{y}{x} = t \Rightarrow y = xt \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$\therefore$ (i) becomes, $t + x\dfrac{dt}{dx} = t + \dfrac{x}{e^t}$

$\Rightarrow \quad x\dfrac{dt}{dx} = xe^{-t} \Rightarrow \dfrac{dt}{dx} = e^{-t} \Rightarrow dx = e^t dt$

Integrating both sides, we get

$x = e^t + C \Rightarrow x = e^{y/x} + C$

**15.** We have, $\dfrac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

$\therefore \dfrac{dy}{dx} = 1 + x^2 + y^2(1 + x^2) = (1 + x^2)(1 + y^2)$

$\Rightarrow \dfrac{dy}{1+y^2} = (1 + x^2)dx$

Integrating both sides, we get $\tan^{-1} y = x + \dfrac{x^3}{3} + C$

when $x = 0, y = 1$

$\therefore \quad \tan^{-1} 1 = 0 + 0 + C \Rightarrow C = \dfrac{\pi}{4}$

$\therefore \tan^{-1} y = x + \dfrac{1}{3}x^3 + \dfrac{\pi}{4}$ is the required solution.

**16.** We have, $(1 - y^2)(1 + \log x)\, dx + 2xy\, dy = 0$

$\Rightarrow \quad (1 - y^2)(1 + \log x)\, dx = -2xy\, dy$

$\Rightarrow \quad \dfrac{(1+\log x)}{x}dx = -\dfrac{2y}{1-y^2}dy$

On integrating both sides, we get

$\dfrac{(1+\log x)^2}{2} = \log|1 - y^2| + C$

When $x = 1, y = 0$

$\therefore \quad \dfrac{(1+\log 1)^2}{2} = \log(1) + C \Rightarrow C = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{(1+\log x)^2}{2} = \log|1 - y^2| + \dfrac{1}{2}$

$\Rightarrow \quad (1 + \log x)^2 = 2\log|1 - y^2| + 1$ is the required particular solution.

**17.** We have, $\dfrac{dy}{dx} = \dfrac{x(2\log x + 1)}{\sin y + y\cos y}$

$\Rightarrow \quad (\sin y + y\cos y)dy = x(2\log x + 1)dx$

On integrating both sides, we get

$-\cos y + y\sin y - (-\cos y)$

$$= 2\left[\log x \times \frac{x^2}{2} - \int \frac{1}{x} \times \frac{x^2}{2}dx\right] + \frac{x^2}{2} + C$$

$\Rightarrow \quad y\sin y = x^2\log x - \dfrac{x^2}{2} + \dfrac{x^2}{2} + C$

$\Rightarrow \quad y\sin y = x^2\log x + C$

When $x = 1, y = \dfrac{\pi}{2}$

$\therefore \dfrac{\pi}{2}\sin\dfrac{\pi}{2} = 1 \cdot \log(1) + C \Rightarrow \dfrac{\pi}{2} = C$

$\therefore \quad y\sin y = x^2\log x + \pi/2$ is the required particular solution.

**18.** We have, $x(1 + y^2)\, dx - y(1 + x^2)\, dy = 0$

$\Rightarrow \quad \dfrac{x}{1+x^2}dx - \dfrac{y}{1+y^2}dy = 0$

$\Rightarrow \quad \dfrac{2x}{1+x^2}dx = \dfrac{2y}{1+y^2}dy$

Integrating both sides, we get

$\log(1 + y^2) = \log(1 + x^2) + \log C$

$\Rightarrow \quad 1 + y^2 = C(1 + x^2)$

When $x = 0, y = 1$

$\therefore \quad 1 + 1 = C(1 + 0) \Rightarrow C = 2$

$\therefore \quad 1 + y^2 = 2(1 + x^2)$ is the required particular solution.

**19.** We have, $xy\dfrac{dy}{dx} = (x + 2)(y + 2)$

$\Rightarrow \quad \dfrac{y\,dy}{(y+2)} = \left(\dfrac{x+2}{x}\right)dx$

$\Rightarrow \quad dy - \dfrac{2}{(y+2)}dy = dx + \dfrac{2}{x}dx$

Integrating both sides, we get

$y - 2\log(y + 2) = x + 2\log x + C$

when $x = 1, y = -1$

So, $-1 - 2\log(-1 + 2) = 1 + 2\log 1 + C$

$\Rightarrow \quad C = -1 - 1 = -2$

So, we have $y - 2\log(y + 2) = x + 2\log x - 2$

$\Rightarrow \quad y - x + 2 = 2\log(x(y + 2))$.

**20.** We have, $(x + 1)\dfrac{dy}{dx} = 2e^{-y} - 1$ ...(i)

$\Rightarrow \quad \dfrac{dy}{2e^{-y} - 1} = \dfrac{dx}{x+1} \Rightarrow \dfrac{e^y}{2 - e^y}dy = \dfrac{dx}{x+1}$

Integrating both sides, we get

$-\log(2 - e^y) = \log(x + 1) + C$ ...(ii)

When $x = 0, y = 0$

$\therefore \quad -\log(2 - 1) = \log(0 + 1) + C \Rightarrow C = 0$

$\therefore \quad$ (ii) becomes

$-\log(2 - e^y) = \log(x + 1)$

$\Rightarrow \quad \log(x + 1) + \log(2 - e^y) = 0$

$\Rightarrow \quad \log[(x + 1)(2 - e^y)] = 0$

$\Rightarrow \quad (x + 1)(2 - e^y) = 1$ is the required particular solution.

**21.** We have $(x + a)^2 + (y - a)^2 = a^2$ ...(i)

which has only one arbitrary constant $a$.

Differentiating (i) w.r.t. $x$, we get

$2(x + a) + 2(y - a)\dfrac{dy}{dx} = 0 \Rightarrow a = \dfrac{x + yy'}{y' - 1}$ ...(ii)

Substituting value of $a$ from (ii) in (i), we get

$$\left(x+\frac{x+y'y}{y'-1}\right)^2+\left(y-\frac{x+y'y}{y'-1}\right)^2=\left(\frac{x+y'y}{y'-1}\right)^2$$

$\Rightarrow [x(y'-1)+x+y'y]^2+[y(y'-1)-x-y'y]^2=(x+y'y)^2$

$\Rightarrow (x+y)^2(y')^2+(x+y)^2=(x+y'y)^2$

$\Rightarrow \ (x+y)^2\left[\left(\frac{dy}{dx}\right)^2+1\right]=\left(x+y\frac{dy}{dx}\right)^2$, is the required differential equation.

**22.** We have $x^2=4ay$, where $a$ is the constant. ...(i)

Differentiating (i) w.r.t. $x$, we get

$2x=4ay_1 \Rightarrow \ \frac{2x}{y_1}=4a$ ...(ii)

Substituting the value of $4a$ from (ii) in (i), we get

$x^2=\frac{2x}{y_1}y \ \Rightarrow x^2y_1-2xy=0 \ \Rightarrow \ xy_1-2y=0$

$\Rightarrow \ x\frac{dy}{dx}-2y=0,$ is the required differential equation.

**23.** We have, $xe^xdy=(x^3+2ye^x)dx$

$\Rightarrow \ \frac{dy}{dx}=\frac{x^3+2ye^x}{xe^x} \ \Rightarrow \ \frac{dy}{dx}-\frac{2}{x}y=x^2e^{-x}$ ...(i)

This is a linear D.E. of the form $\frac{dy}{dx}+Py=Q$

$\therefore$ I.F. $=e^{-\int\frac{2}{x}dx}=e^{-2\log x}=e^{\log x^{-2}}=\frac{1}{x^2}$

So, the solution of (i) is

$$y\cdot\frac{1}{x^2}=\int\frac{1}{x^2}\cdot x^2e^{-x}dx$$

$\Rightarrow \ \frac{y}{x^2}=-e^{-x}+C \ \Rightarrow \ y=-x^2e^{-x}+Cx^2$

which is the required solution.

**24.** We have, $x\frac{dy}{dx}=y-x\tan\left(\frac{y}{x}\right)$

$\Rightarrow \ \frac{dy}{dx}=\frac{y}{x}-\tan\left(\frac{y}{x}\right),$

which is a homogeneous differential equation.

Now, put $y=vx \ \Rightarrow \ \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore \ v+x\frac{dv}{dx}=v-\tan v$

$\Rightarrow \ x\frac{dv}{dx}=-\tan v \ \Rightarrow \ \frac{dv}{\tan v}=-\frac{dx}{x}$

$\Rightarrow \ \cot v\,dv+\frac{dx}{x}=0$

Integrating both sides,we get

$\log|\sin v|+\log x=\log C$

$\Rightarrow \ x\sin v=C \ \Rightarrow \ x\sin\left(\frac{y}{x}\right)=C$

When $x=1$, $y=\frac{\pi}{4}$

$1\cdot\sin\left(\frac{\pi}{4}\right)=C \ \Rightarrow \ C=\frac{1}{\sqrt{2}}$

So, $x\sin\left(\frac{y}{x}\right)=\frac{1}{\sqrt{2}}$ is the required particular solution.

**25.** We have, $x\frac{dy}{dx}-y=\sqrt{x^2+y^2}$

$\Rightarrow \ \frac{dy}{dx}=\frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2}$ ...(i)

This is a homogeneous differential equation.

Put $y=vx \ \Rightarrow \ \frac{dy}{dx}=v+x\frac{dv}{dx}$

$\therefore$ (i) becomes

$v+x\frac{dv}{dx}=v+\sqrt{1+v^2} \Rightarrow x\frac{dv}{dx}=\sqrt{1+v^2}$

$\Rightarrow \ \frac{dx}{x}=\frac{dv}{\sqrt{1+v^2}} \ \Rightarrow \int\frac{dx}{x}=\int\frac{dv}{\sqrt{1+v^2}}$

$\Rightarrow \ \log x+\log C_1=\log|v+\sqrt{1+v^2}|$

$\Rightarrow \ \log x+\log C_1=\log\left|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right|$

$\Rightarrow \ \log C_1x=\log|y+\sqrt{x^2+y^2}|-\log x$

$\Rightarrow \ \pm C_1x^2=y+\sqrt{x^2+y^2}$

$\Rightarrow \ Cx^2=y+\sqrt{x^2+y^2}$ [where $C=\pm C_1$]

When $x=1$, $y=0$

$\therefore \ C=0+\sqrt{1+0}\Rightarrow C=1$

$\therefore$ Required particular solution is $x^2=y+\sqrt{x^2+y^2}$.

**26.** We have $(1+x^2)\frac{dy}{dx}+2xy=4x^2$

$\Rightarrow \ \frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}$

This is a linear differential equation of the form

$\frac{dy}{dx}+Py=Q$ where $P=\frac{2x}{1+x^2}$ and $Q=\frac{4x^2}{1+x^2}$

$\therefore$ I.F. $=e^{\int Pdx}=e^{\int\frac{2x}{1+x^2}dx}$

$=e^{\log(1+x^2)}=1+x^2$

Hence, the required solution is

$$y(1+x^2)=\int\frac{4x^2}{1+x^2}(1+x^2)\,dx+C$$

$\Rightarrow \ y(1+x^2)=4\int x^2dx+C$

$\Rightarrow \quad y(1+x^2) = \dfrac{4x^3}{3} + C$

Given that $y(0) = 0$

$\therefore \quad 0(1 + 0) = 0 + C \Rightarrow C = 0$

Thus, $y = \dfrac{4x^3}{3(1+x^2)}$ is the required solution.

**27.** We have, $\dfrac{dy}{dx} = \dfrac{xy}{x^2+y^2}$

This is a homogeneous differential equation

$\therefore \quad$ Put $y = vx \;\Rightarrow\; \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore \quad v + x\dfrac{dv}{dx} = \dfrac{x \cdot vx}{x^2+v^2x^2}$

$\Rightarrow \quad v + x\dfrac{dv}{dx} = \dfrac{v}{1+v^2} \;\Rightarrow\; x\dfrac{dv}{dx} = \dfrac{v}{1+v^2} - v$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{-v^3}{1+v^2} \quad\Rightarrow\quad \dfrac{dx}{x} = -\left(\dfrac{1+v^2}{v^3}\right)dv$

Integrating both sides, we get

$\displaystyle\int \frac{dx}{x} = -\int v^{-3}dv - \int \frac{1}{v}dv$

$\Rightarrow \log x = \dfrac{1}{2v^2} - \log v + C$

$\Rightarrow \quad \log x = \dfrac{x^2}{2y^2} - \log y + \log x + C$

$\Rightarrow \quad \log y = \dfrac{x^2}{2y^2} + C$

When $x = 0, y = 1 \;\Rightarrow\; \log 1 = 0 + C \;\Rightarrow\; C = 0$

$\therefore \quad$ Particular solution is $\log y = \dfrac{x^2}{2y^2} \Rightarrow y = e^{\frac{x^2}{2y^2}}$

**28.** The given differential equation is,

$e^x \tan y\, dx + (2 - e^x) \sec^2 y\, dy = 0$

$\Rightarrow \quad (2 - e^x)\sec^2 y\, dy = -e^x \tan y\, dx$

$\Rightarrow \quad \dfrac{\sec^2 y}{\tan y}dy = \dfrac{-e^x}{2-e^x}dx$

Integrating both sides, we get

$\displaystyle\int \frac{\sec^2 y}{\tan y}dy = \int \frac{-e^x}{2-e^x}dx$

$\Rightarrow \quad \log \tan y = \log(2 - e^x) + C$

When $x = 0, y = \dfrac{\pi}{4}$

$\therefore \; \log\tan\dfrac{\pi}{4} = \log(2-e^0) + C$

$\Rightarrow \quad 0 = \log 1 + C \Rightarrow C = 0$

$\therefore \quad$ Particular solution is $\log \tan y = \log (2 - e^x)$

*i.e.*, $e^x + \tan y - 2 = 0$

**29.** We have, $y^2\, dx + (x^2 - xy + y^2)\, dy = 0$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{-y^2}{x^2-xy+y^2}$

This is a homogeneous differential equation.

$\therefore\;$ Put $y = vx \;\Rightarrow\; \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$, we get

$v + x\dfrac{dv}{dx} = \dfrac{-v^2x^2}{x^2 - vx^2 + v^2x^2}$

$\Rightarrow \quad v + x\dfrac{dv}{dx} = \dfrac{-v^2}{1-v+v^2}$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{-v^2}{1-v+v^2} - v$

$\Rightarrow \quad x\dfrac{dv}{dx} = \dfrac{-v-v^3}{1-v+v^2} \;\Rightarrow\; \dfrac{1-v+v^2}{v(1+v^2)}dv = -\dfrac{1}{x}dx$

Integrating both sides, we get

$\displaystyle\int \frac{1+v^2}{v(1+v^2)}dv - \int \frac{v}{v(1+v^2)}dv = -\int \frac{1}{x}dx$

$\Rightarrow \quad \displaystyle\int \frac{1}{v}dv - \int \frac{1}{1+v^2}dv = -\int \frac{1}{x}dx$

$\Rightarrow \quad \log |v| - \tan^{-1}v = -\log |x| + \log C$

$\Rightarrow \quad \log\left|\dfrac{vx}{C}\right| = \tan^{-1}v \;\Rightarrow\; \left|\dfrac{vx}{C}\right| = e^{\tan^{-1}v}$

$\Rightarrow \quad |y| = Ce^{\tan^{-1}(y/x)}$ is the required solution.

**30.** We have, $(x^2-1)\dfrac{dy}{dx} + 2xy = \dfrac{2}{x^2-1}, |x| \neq 1$

$\Rightarrow \quad \dfrac{dy}{dx} + \dfrac{2xy}{x^2-1} = \dfrac{2}{(x^2-1)^2}$

This is a linear differential equation of the form,

$\dfrac{dy}{dx} + Py = Q$, where $P = \dfrac{2x}{x^2-1}$ and $Q = \dfrac{2}{(x^2-1)^2}$

$\therefore\;$ I.F. $= e^{\int P dx} = e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2 - 1$

Hence, solution of the differential equation is given by

$y(x^2-1) = \displaystyle\int \frac{2(x^2-1)}{(x^2-1)^2}dx$

$\Rightarrow \quad y(x^2-1) = 2\displaystyle\int \frac{dx}{x^2-1}$

$\Rightarrow \quad y(x^2-1) = 2 \times \dfrac{1}{2}\log\left|\dfrac{x-1}{x+1}\right| + C$

$\Rightarrow \quad y(x^2-1) = \log\left|\dfrac{x-1}{x+1}\right| + C$

**31.** We have, $\frac{dy}{dx}=1+x+y+xy$

$\Rightarrow \frac{dy}{dx}=(1+x)+(1+x)y$ = $(1+x)(1+y)$

$\Rightarrow \frac{dy}{1+y}=(1+x)dx$

Integrating both sides, we get

$\int\frac{dy}{1+y}=\int(1+x)dx+C$

$\Rightarrow \log(1+y)=x+\frac{x^2}{2}+C$ ...(i)

When $x = 1, y = 0$

$\therefore \log 1=1+\frac{1}{2}+C \Rightarrow C=-\frac{3}{2}$

$\therefore$ The particular solution is

$\log(1+y)=x+\frac{x^2}{2}-\frac{3}{2}.$

**32.** We have, $\frac{dy}{dx}+y\cot x=2\cos x$

This is a linear differential equation of the form $\frac{dy}{dx}+Py=Q$, where $P = \cot x$, $Q = 2\cos x$

$\therefore$ I.F. $=e^{\int\cot x\,dx}=e^{\log|\sin x|}=|\sin x|$

$\therefore y|\sin x| = \int|\sin x|(2\cos x)dx$

$\Rightarrow y|\sin x|=\int\sin 2x dx$

$\Rightarrow y(\sin x)=-\frac{1}{2}\cos 2x+C$

When $x=\frac{\pi}{2}$, $y=0$

$0(\sin\frac{\pi}{2})=-\frac{1}{2}\cos 2\left(\frac{\pi}{2}\right)+C\Rightarrow C=-\frac{1}{2}$

$\therefore y(\sin x)=-\frac{1}{2}\cos 2x-\frac{1}{2}$

*i.e.*, $2y\sin x+\cos 2x+1=0$ is the required solution.

**33.** We have, $\left(\frac{2+\sin x}{1+y}\right)\frac{dy}{dx}=-\cos x$

$\Rightarrow \frac{dy}{1+y}=-\frac{\cos x}{2+\sin x}dx$

Integrating both sides, we get

$\log(y+1) = -\log|2+\sin x| + \log C$

$\Rightarrow \log(y+1)=\log\left(\frac{C}{2+\sin x}\right)$

$\Rightarrow y+1 = \frac{C}{2+\sin x} \Rightarrow (y+1)(2+\sin x) = C$

Given : $y(0) = 1 \Rightarrow x = 0, y = 1$

$\therefore (1+1)(2+\sin 0) = C \Rightarrow C = 4$

$\therefore (y+1)(2+\sin x) = 4$

$\Rightarrow y=\frac{4}{2+\sin x}-1$ ...(i)

Put $x=\frac{\pi}{2}$ in (i), $y\left(\frac{\pi}{2}\right)=\frac{4}{2+1}-1=\frac{1}{3}.$

**34.** We have $x\frac{dy}{dx}-y+x\sin\left(\frac{y}{x}\right)=0$

$\Rightarrow \frac{dy}{dx}-\frac{y}{x}+\sin\left(\frac{y}{x}\right)=0$

This is a linear homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx}=v\cdot 1+x\frac{dv}{dx}$

$\therefore v+x\frac{dv}{dx}-v+\sin v=0$

$\Rightarrow x\frac{dv}{dx}+\sin v=0 \Rightarrow \operatorname{cosec} v\,dv+\frac{dx}{x}=0$

Integrating both sides, we get

$\log|\operatorname{cosec} v-\cot v|+\log x=\log C$

$\Rightarrow x(\operatorname{cosec} v-\cot v)=C$

$\Rightarrow x\left[\operatorname{cosec}\left(\frac{y}{x}\right)-\cot\left(\frac{y}{x}\right)\right]=C$

when $x = 2, y = \pi$

$\therefore 2\left[\operatorname{cosec}\frac{\pi}{2}-\cot\frac{\pi}{2}\right]=C\Rightarrow C=2$

$\Rightarrow x\left[\operatorname{cosec}\left(\frac{y}{x}\right)-\cot\left(\frac{y}{x}\right)\right]=2$ is the required particular solution.

**35.** We have, $\frac{dy}{dx}=\frac{y^2}{xy-x^2}=\frac{y^2/x^2}{(xy-x^2)/x^2}$ ... (i)

$\Rightarrow \frac{dy}{dx}=\frac{y^2/x^2}{\frac{y}{x}-1}$

$\Rightarrow$ It is a homogeneous differential equation

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx}=v\cdot 1+x\frac{dv}{dx}$

$\therefore$ (i) becomes

$v+x\frac{dv}{dx}=\frac{v^2}{v-1} \Rightarrow x\frac{dv}{dx}=\frac{v^2}{v-1}-v\Rightarrow x\frac{dv}{dx}=\frac{v}{v-1}$

$\Rightarrow \frac{v-1}{v}dv=\frac{dx}{x} \Rightarrow \left(1-\frac{1}{v}\right)dv=\frac{dx}{x}$

Integrating both sides, we get

$v-\log v=\log x+C \Rightarrow v=\log vx+C$

$\Rightarrow \frac{y}{x}=\log y+C$

$\Rightarrow y=x(\log y+C)$ is the required solution.

**36.** We have, $(x-y)\frac{dy}{dx}=x+2y$

$\frac{dy}{dx}=\frac{x+2y}{x-y}$ ...(i)

Put $y = Vx \quad \Rightarrow \quad \frac{dy}{dx} = V + x\frac{dV}{dx}$

Putting $\frac{dy}{dx} = V + x\frac{dV}{dx}$ in (i), we get

$$V + x\frac{dV}{dx} = \frac{x+2Vx}{x-Vx} \Rightarrow V + x\frac{dV}{dx} = \frac{1+2V}{1-V}$$

$$\Rightarrow x\frac{dV}{dx} = \frac{1+2V}{1-V} - V \Rightarrow x\frac{dV}{dx} = \frac{1+2V-V+V^2}{1-V}$$

$$\Rightarrow \int\frac{1-V}{V^2+V+1}dV = \int\frac{1}{x}dx$$

$$\Rightarrow \int\frac{2-2V}{V^2+V+1}dV = 2\log|x| + c$$

$$\Rightarrow \int\frac{3-(2V+1)}{V^2+V+1}dV = 2\log|x| + c$$

$$\Rightarrow \int\frac{3}{V^2+V+1}dV - \int\frac{2V+1}{V^2+V+1}dV = \log|x|^2 + c$$

$$\Rightarrow 3\int\frac{1}{\left(V+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dV - \log|V^2+V+1|$$

$$= \log\ |x^2|\ + c$$

$$\Rightarrow \frac{3}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left(\frac{V+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \log\ |x^2\,(V^2+V+1)|\ + c$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\left(\frac{2V+1}{\sqrt{3}}\right) = \log|x^2(V^2+V+1)| + c \qquad \text{...(ii)}$$

Substituting $V = \frac{y}{x}$ in (ii), we get

$$2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \log\left|x^2\frac{(y^2+yx+x^2)}{x^2}\right| + c$$

$$\Rightarrow 2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \log(y^2+xy+x^2) + c$$

Now, at $y = 0$ and $x = 1$, we have

$$2\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \log|1| + c$$

$$\Rightarrow c = 2\sqrt{3}\cdot\frac{\pi}{6} = \frac{\pi}{\sqrt{3}}$$

Substituting $c = \frac{\pi}{\sqrt{3}}$ in (iii), we get

$$2\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \log|y^2+xy+x^2| + \frac{\pi}{\sqrt{3}}$$

$$\Rightarrow 6\tan^{-1}\left(\frac{2y+x}{\sqrt{3}x}\right) = \sqrt{3}\log(x^2+xy+y^2) + \pi$$

**37.** We have, $(\tan^{-1}x - y)dx = (1+x^2)dy$

$$\Rightarrow \frac{dy}{dx} = \frac{\tan^{-1}x - y}{1+x^2} \Rightarrow \frac{dy}{dx} + \frac{1}{1+x^2}\cdot y = \frac{\tan^{-1}x}{1+x^2}$$

This is a linear differential equation of the form

$\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{1+x^2}$ and $Q = \frac{\tan^{-1}x}{1+x^2}$

$\therefore$ I.F. $= e^{\int\frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$

$\therefore$ Solution is given by

$$ye^{\tan^{-1}x} = \int e^{\tan^{-1}x}\left(\frac{\tan^{-1}x}{1+x^2}\right)dx + C$$

$$\Rightarrow ye^{\tan^{-1}x} = e^{\tan^{-1}x}\cdot\tan^{-1}x - e^{\tan^{-1}x} + C$$

$$y = \tan^{-1}x - 1 + Ce^{-\tan^{-1}x} \qquad \text{...(i)}$$

Now, putting $x = 0$, $y = 1$ in (i), we get

$1 = \tan^{-1}0 - 1 + Ce^{-\tan^{-1}(0)} \Rightarrow C = 2$

So, required particular solution is

$y = \tan^{-1}x - 1 + 2e^{-\tan^{-1}x}$.

**38.** We have, $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{(1+x^2)(1+y^2)}}{xy}$$

$$\Rightarrow \int\frac{y}{\sqrt{1+y^2}}dy = -\int\frac{\sqrt{1+x^2}}{x^2}xdx$$

$$\Rightarrow \frac{1}{2}\int\frac{2y}{\sqrt{1+y^2}}dy = -\int\frac{v^2}{v^2-1}dv$$

[putting $1 + x^2 = v^2 \Rightarrow 2xdx = 2vdv$]

$$\Rightarrow \sqrt{1+y^2} = -\int\left(1+\frac{1}{v^2-1}\right)dv$$

$$\Rightarrow \sqrt{1+y^2} = -v - \frac{1}{2}\log\left|\frac{v-1}{v+1}\right| + C$$

$$\Rightarrow \sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = C$$

**39.** We have, $(x-y)\frac{dy}{dx} = x+2y$

$$\Rightarrow \frac{dy}{dx} = \frac{x+2y}{x-y} \qquad \text{...(i)}$$

This is a linear homogeneous differential equation.

$\therefore$ Put $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\frac{dv}{dx}$

$\therefore$ (i) becomes

$$v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx} = \frac{1+2v}{1-v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+v+v^2}{1-v}$$

$$\Rightarrow \frac{1-v}{1+v+v^2}dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{-\frac{1}{2}(2v+1)+\frac{3}{2}}{v^2+v+1}dv = \log x + C$$

$$\Rightarrow \quad -\frac{1}{2}\int \frac{2v+1}{v^2+v+1}dv + \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$= \log x + C$

$$\Rightarrow \quad -\frac{1}{2}\log\left(v^2+v+1\right) + \frac{3}{2}\cdot\frac{1}{\sqrt{3}/2}\tan^{-1}\left[\frac{v+\frac{1}{2}}{\sqrt{3}/2}\right] = \log x + C$$

$$\Rightarrow \ -\frac{1}{2}\log\left(\frac{y^2}{x^2}+\frac{y}{x}+1\right)+\sqrt{3}\tan^{-1}\left(\frac{2y+x}{\sqrt{3}\cdot x}\right) = \log x + C$$

The general solution is

$$\log x + C = \frac{-1}{2}\log\left(\frac{y^2}{x^2}+\frac{y}{x}+1\right) + \sqrt{3}\tan^{-1}\left[\left(\frac{2y}{x}+1\right)/\sqrt{3}\right] \quad \text{...(i)}$$

Putting $x = 1$, $y = 0$ in (i), we get

$$0 + C = -\frac{1}{2}\log(0+0+1)+\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \ \Rightarrow \ C = \frac{\pi}{2\sqrt{3}}$$

$$\therefore \quad \log x + \frac{\pi}{2\sqrt{3}} = -\frac{1}{2}[\log(y^2+xy+x^2)-\log x^2] + \sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right)$$

$$\Rightarrow \quad \frac{\pi}{2\sqrt{3}} = -\frac{1}{2}\log(x^2+xy+y^2)+\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}x}\right)$$

**40.** We have, $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^3-3xy^2}{y^3-3x^2y} \quad \text{...(i)}$$

Put, $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$\therefore$ (i) becomes

$$v + x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1-3v^2-v^4+3v^2}{v^3-3v} \Rightarrow x\frac{dv}{dx} = \frac{1-v^4}{v(v^2-3)}$$

$$\Rightarrow \quad \frac{v(v^2-3)dv}{1-v^4} = \frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{(v^3-3v)dv}{(1-v^2)(1+v^2)} = \int \frac{dx}{x} \quad \text{...(ii)}$$

Now, let $\dfrac{v^3-3v}{(1-v^2)(1+v^2)} = \dfrac{Av+B}{1-v^2} + \dfrac{Cv+D}{1+v^2}$ ...(iii)

$\Rightarrow \quad v^3 - 3v = (Av + B)(1 + v^2) + (Cv + D)(1 - v^2)$

Comparing coeff. of like powers, we get

$A - C = 1$, $A + C = -3$, $B - D = 0$ and $B + D = 0$

Solving these equations, we get $A = -1$, $B = 0$, $C = -2$, $D = 0$ ...(iv)

From (ii), (iii) and (iv), we have

$$\int \frac{-v}{1-v^2}dv - \int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \frac{1}{2}\log(1-v^2) - \log(1+v^2) = \log x + \log C_1$$

$$\Rightarrow \quad \frac{\sqrt{1-v^2}}{1+v^2} = C_1 x \ \Rightarrow \ x\frac{\left(\sqrt{x^2-y^2}\right)}{x^2+y^2} = C_1 x$$

$\Rightarrow \quad x^2 - y^2 = C_1^2(x^2 + y^2)^2$

*i.e.*, $x^2 - y^2 = C(x^2 + y^2)^2$ (where $C_1^2 = C$)

which is the required solution.

Hence Proved.